MT 1500 ALGEBRA, ANAL GEOM, CAL. & Tring.

PART – A

Answer ALL the questions **(10 x 2 = 20 marks)**

1. **Find the nth derivative of $\sin^3 2x$.**
2. **Show that in the parabola $y^2 = 4ax$, the subtangent at any point is double the abscissa.**
3. **Find the radius of curvature of $x^4 + y^4 = 2$ at (1, 1).**
4. **Give the coordinates of the centre of curve line at any point.**
5. **Form an equation with rational coefficients having $\sqrt{2} + 1$ as a root.**
6. **If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ then find the value of $\propto^2 + \beta^2 + \gamma^2$.**
7. **Show that $1 - \tanh^2 x = \text{Sech}^2 x$.**
8. **Evaluate $\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 2\theta}$.**
9. **Find the polar of the point (1, 2) on $y^2 = 4x$.**
10. **Give the condition of the diameters $y = m_1 x$ and $y = m_2 x$ of an ellipse to be conjugate.**

PART – B

Answer any FIVE questions **(5 x 8 = 40 marks)**

11. **State and prove Leibnitz theorem on the nth derivative of a product of two functions.**
12. **Show that in the curve $r = ae^{\theta \cot \alpha}$, the tangent is inclined at a constant angle to the raduis vector.**
13. **Find the minimum value of $x^2 + 5y^2 - 6x + 10y + 12$.**
14. **Find the raduis of curvature at the point θ on $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$.**
15. **Show that if the roots of $x^3 + px^2 + qx + r = 0$ are in A.P. then $2p^3 - 9pq + 27r = 0$.**
16. **Solve $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + i\sqrt{3}$ is a root.**
17. **Expand $\sin^3\theta \cos^4\theta$ in terms of sines of multiples of θ.**
18. **If P and D are the extremities of a pair of conjugate diameters of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, find the locus of the midpoint of PD.**

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. a) If $y = \left[\log(x + \sqrt{x^2 + 1})\right]^2$ then show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$.

b) Show that r = a $\sec^2\frac{\theta}{2}$ $and\ r = b\ cosec^2\frac{\theta}{2}$ intersect at right angles. **(10+10)**

20. a) Find the maximum and minimum of $3x^2 + 4y^2 - xy$ if $2x + y = 21$.

b) Find the p – r equation of $\frac{2a}{r} = 1 - \cos\theta$ with respect to the focus as pole.

(10 + 10)

21. a) Solve: $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

b) Find the positive root of $x^3 + 24x = 50$ to two places of decimals using Horner's method. **(10 + 10)**

22. a) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$ then show that $\cos 2\theta \cosh 2\phi = 3$.

b) Show that the locus of poles with respect to $y^2 = 4ax$ of tangents to $x^2 - y^2 = a^2$ is an ellipse. **(10+10)**

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